UNIQUE FEATURES OF HEAT EXCHANGE FOR SPHERICAL PARTICLES IN A CONCENTRIC CAVITY

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Exact and general solutions are obtained for the problem of radiant heat exchange of a sphere in a cavity. Radiation effects in such a system are analyzed.

Techniques for high output production of monodispersed spherical particles and the use of such objects in a number of promising technological applications raises the question of study of the fundamental physical properties of microparticles, in particular, their radiant thermal characteristics. Since in the behavior of small size particles fine-scale effects which are usually neglected in the study of larger objects play a marked role, a sequential properly structured analysis of typical heat exchange situations is necessary. In connection with this it is of interest to consider the emission properties of a particle located in the center of an evacuated cavity, a situation often found in experimental practice (Fig. 1). Analytical determination of the exact value of the radiant flux in such a gap is a quite complex problem and is normally based on the fluctuation method proposed in [1]. Omitting a description of the process of finding this solution, we will only present its final form:

$$P_\omega = e_\omega \pi \Delta I_{0,\omega}, \tag{1}$$

$$\Delta I_{0,\omega} = \frac{\hbar\omega^3}{4\pi^3 c^2}\left(\left(\exp\left(\frac{\hbar\omega}{k_B T}\right)-1\right)^{-1} - \left(\exp\left(\frac{\hbar\omega}{k_B T_1}\right)-1\right)^{-1}\right), \tag{2}$$

$$e_\omega = \frac{1}{2x^6\gamma^2}\sum_{m=1}^{\infty}(2m+1)\left(\frac{S_{\mu,m}S^{(0)}_{\mu,m}}{|F_{1,m}|^2} + \frac{S_{\varepsilon,m}S^{(0)}_{\varepsilon,m}}{|F_{2,m}|^2}\right). \tag{3}$$

Here we have introduced the following notation and abbreviations:

$$x = \omega r/c;\ \gamma = r_c/r;\ y = x\gamma;\ \rho = x\sqrt{\varepsilon\mu};\ \xi = \sqrt{\mu/\varepsilon};\ \xi_1 = \sqrt{\mu_1/\varepsilon_1};$$

$$S_{\mu,m} = \frac{1}{\xi}\frac{\partial}{\partial\rho}\ln\rho j_{\rho,m} - \text{c.c};\ S_{\varepsilon,m} = \frac{1}{\xi^*}\frac{\partial}{\partial\rho}\ln\rho j_{\rho,m} - \text{c.c.};$$

$$S^{(0)}_{\mu,m} = \frac{1}{\xi_1}\frac{\partial}{\partial\rho_1}\ln\rho_1 h_{\rho_1,m} - \text{c.c.};\ S^{(0)}_{\varepsilon,m} = \frac{1}{\xi_1^*}\frac{\partial}{\partial\rho_1}\ln\rho_1 h_{\rho_1,m} - \text{c.c.};$$

$$F_1 = j_y n_x \delta_{j,n}\delta^{(0)}_{h,j} - j_x n_y \delta_{j,j}\delta^{(0)}_{h,n};\ F_2 = j_y n_x \Delta^{(0)}_{j,h}\Delta_{n,j} - j_x n_y \Delta_{j,j}\Delta^{(0)}_{n,h};$$

$$\delta_{f,\varphi} = \frac{1}{\xi}\frac{\partial}{\partial\rho}\ln\rho f_\rho - \frac{\partial}{\partial x}\ln x\varphi_x;\ \Delta_{f,\varphi} = \frac{1}{\xi}\frac{\partial}{\partial x}\ln x f_x - \frac{\partial}{\partial\rho}\ln\rho\varphi_\rho;$$

$$\Delta^{(0)}_{f,\varphi} = \frac{1}{\xi_1}\frac{\partial}{\partial y}\ln y f_y - \frac{\partial}{\partial\rho_1}\ln\rho_1\varphi_{\rho_1};\ \delta^{(0)}_{f,\varphi} = \frac{1}{\xi_1}\frac{\partial}{\partial\rho_1}\ln\rho_1 f_{\rho_1} - \frac{\partial}{\partial y}\ln y\varphi_y$$

(c.c. and * denote complex conjugates).

Equations (1)-(3) completely describe radiant heat exchange in a spherical vacuum gap for arbitrary dimensions and parameters of the media participating in heat exchange. If instead of vacuum the gap is filled with another, but also transparent, medium (i.e., with real ε_S, μ_S), then in Eqs. (1)-(3) ξ_1, ξ and the entire expression (3) should be multiplied by $(\varepsilon_S/\mu_S)^{1/2}$ and $(\varepsilon_S \cdot \mu_S)^{-2}$, respectively.

We note that as follows from equations (1)-(3) in the case where the inner sphere is transparent (ε, μ – real), heat exchange is absent regardless of the choice of cavity material (S_μ, S_ε vanish identically), while in the situation with a diathermal cavity (ε_1, μ_1 – real) the flux does not vanish (S^0_μ, S^0_ε do not vanish due to the presence of the com-

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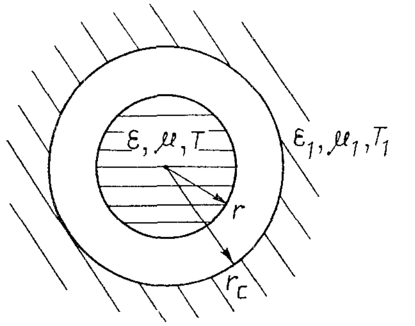

Fig. 1. Heat exchange system: sphere – vacuum gap – concentric cavity.

plex function h = j − in, where i is the imaginary unit), i.e., the sphere radiates to "infinity."

It is of definite interest to study the behavior of series (3) at $r_c = r$. For this purpose we must take the asymptotes of the functions in Eq. (3) as $m \to \infty$ and determine the conditions for convergence. Making the necessary calculations we find that in order of magnitude as $m \to \infty$

$$F_1,\ F_2 \sim m;\quad S_{\mu,m}S^{(0)}_{\mu,m} \sim m^2 I_m(\mu)\, I_m(\mu_1),\quad S_{\varepsilon,m}S^{(0)}_{\varepsilon,m} \sim m^2 I_m(\varepsilon)\, I_m(\varepsilon_1),$$

while the first of these relationships is satisfied only when both corresponding heat exchange parameters ε, ε_1 and μ, μ_1 of the media differ. In the opposite case, i.e., where $\varepsilon = \varepsilon_1$ or $\mu = \mu_1$, the function F_2 (or F_1) vanishes identically as $m \to \infty$ and the series formally diverges ($e_\omega \to \infty$), which is a consequence of the choice of the local dispersion dependence $\varepsilon(\omega)$. If $\varepsilon \neq \varepsilon_1$ and $\mu \neq \mu_1$, then the divergence at $r = r_c$ of series (3) appears in the situation where both media simultaneously have dissipative electrical or magnetic properties, i.e., imaginary components of either ε, ε_1 or μ, μ_1 or all four parameters are simultaneously nonzero. If the dissipative properties of both the electric and the magnetic sort disappear from the system due to one or both media the divergence is eliminated. However in this case although e_ω does not formally increase without limit, it does increase significantly as $r_c \to r$. Convergence of the series at $r_c \neq r$ is insured due to the presence in the asymptotic expansions F_1 and F_2 of the factor $(r_c/r)^{-m}$. Abrupt increase in the intensity of heat exchange for all frequencies upon reduction in the spherical gap to the point of tangency is to be expected in principle, since any closely spaced curved surfaces can be considered planoparallel over small sections, and for such planar structures the radiation intensification effect is well known [2, 3].

We will now consider the question of presence of radiation resonance in heat exchange through a spherical gap for the case of a small Rayleigh sphere, i.e., for $x \ll 1$; $\rho \ll 1$. Performing the required asymptotic expansions and simplifying, we find the condition for vanishing of $F_{1,\,m}$:

$$\frac{n_y}{j_y}\,\frac{\delta^{(0)}_{h,n}}{\delta^{(0)}_{h,j}} = \frac{(2m+1)!!\ (2m-1)!!\ (m\mu+m+1)}{(m+1)(\mu-1)\,x^{2m+1}} \tag{4}$$

and a similar condition for $F_{2,\,m}$:

$$\frac{n_y}{j_y}\,\frac{\Delta^{(0)}_{n,h}}{\Delta^{(0)}_{j,h}} = \frac{(2m+1)!!\,(2m-1)!!\,(m\varepsilon+m+1)}{(m+1)(\varepsilon-1)\,x^{2m+1}} \tag{5}$$

(in equations (4), (5) the m summation index is omitted in the functions on the left sides).

Knowing the concrete dispersion function $\varepsilon(\omega)$, $\mu(\omega)$, $\varepsilon_1(\omega)$, $\mu_1(\omega)$, from Eqs. (4), (5) we can find the exact positions of spectral resonances. As $x \to 0$ solutions (4), (5) contain within themselves together with the single frequency condition for appearance of surface resonance of the internal sphere $\varepsilon(\omega) = -(m+1)/m$ [or $\mu(\omega) = -(m+1)/m$] yet another collective condition for matched interference excitation of the interacting cavity and sphere. The explicit and simple form of this collective resonance will be obtained below, after performing a number of numerical calculations, since direct analytical analysis of Eqs. (4), (5) is quite difficult. We can make some preliminary remarks as to factors affecting collective modes by evaluating the behavior of Eqs. (4), (5) at large $y(\gamma)$, ρ_1. There then appear within the expressions trigonometric (sin, cos) functions of y with complex weight coefficients dependent on ε_1, μ_1. This indicates that spectral positions of collective thermal modes are a periodic series with a strong dependence of excitation intensity on permittivity of the cavity and sphere. Below we will confirm this proposition numerically.

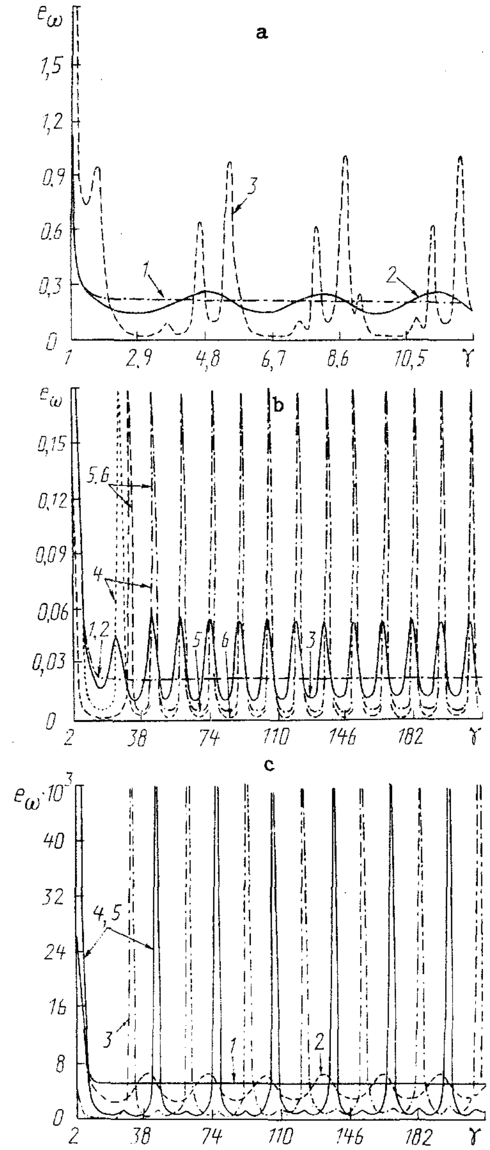

Fig. 2. Spectral coefficient of sphere thermal radiation into cavity: a) B = 10^3, x = 1; b) B = 10, x = 0.1; c) B = 10^5, x = 0.1; cavity parameters: a) $\varepsilon_1=1$ *(1)*, $1-i$ *(2)*, $1-10^2 i$ *(3)*; b) $\varepsilon_1=1$ *(1)*, $1-0{,}1i$ (2), $1-10i$ *(3)*, $1-10^2 i$ *(4)*, $1-10^3 i$ *(5)*, $10^{-2}-10^{-2}i$ *(6)*; c) $\varepsilon_1=1$ *(1)*, $1-i$ *(2)*, $10^{-2}-10^{-2}i$ *(3)*, $1-10^2 i$ *(4)*, $1-10^3 i$ *(5)*.

The appearance of matched excitation upon sphere radiation into the surrounding cavity is completely explicable on the basis of the wave concept of heat transport. The Rayleigh sphere can be considered a point object compared to the wavelength λ, $r \ll \lambda$. For reciprocal radiation of the sphere and cavity one to the other some interference pattern will be formed, which will have a more clearly expressed character at half-wave (or wave) integral conditions known from diffraction theory for radiation to the wall (cavity), while the choice of full or half wavelength is determined by the boundary matching conditions for electromagnetic fields on the wall (i.e., the minimum of the parameters ε_1, μ_1). Nevertheless in the given case we are dealing not with pure interference of wave fields in the normal meaning of the term, since conventional interference and diffraction are a spatial (over angles) distribution of field minima and maxima. In our case we have alternating maxima and minima of a radially directed thermal flux with change in the gap value γ.

Equation (3) was calculated by computer assuming a nonmagnetic sphere and cavity, i.e., for $\mu(\omega) = \mu_1(\omega) = 1$, with the approximation $\varepsilon(\omega)=1-iB/\hat{x}$, $B=4\pi\varkappa r/c$. Some of the results obtained for the sphere radiation coefficient into the cavity as a function of gap size are shown in Fig. 2. Analysis of the numerical data permits the following conclusions.

1. The above proposition of presence of an abrupt increase in spectral flux at any frequency, and, consequently, the full integral flux at limitingly small gap values x is confirmed. For any values of sphere and cavity thermophysical parameters there is an obligatory intensification of heat exchange as the objects approach tangency, but the critical distance γ_{cr} beginning at which this phenomenon is observed, depends to some degree on the parameters x, B, ε_1, although it may be estimated approximately as $\Delta r_{cr}=r_c-r\approx 0{,}1\lambda_W(T_{min})$, where $\lambda_W(T_{min})$

is the Wien wavelength at the temperature of the colder body, which agrees completely with the data of [4, 5]. The physical justification of this situation consists of the following: according ot the conclusions of [1], near any heated body there exists some layer, the "thermal coat" of high thermal energy density created by the steady state component and standing waves of the fluctuation field. As two objects approach, beginning at some distance their "thermal coats" begin interacting and readjust themselves to the newly created situation. A portion of the standing modes having very high fluctuation field electromagnetic energy density tunnel and are now distributed within the closely neighboring bodies, intensifying their responses to an external perturbation, which leads to subsequent intensification of heat liberation.

2. As is evident from Fig. 2, in the region of small diffraction parameters ($x \lesssim 10$) the character of the change in e_ω is close to ideally periodic, differing totally from the behavior of the classical monotonic dependence. For $x > 10$ the "ripple" predominates in the behavior of $e_\omega(\gamma)$ due to excitation of modes of Eq. (3) of higher order than $m = 1$. It is important to note that the mean value of $\bar{e}_\omega$ about which $e_\omega(\gamma)$ varies periodically as $\gamma \to \infty$ is determined not only by the parameters of the inner sphere but also the parameters of the surrounding cavity. This is especially significant when the characteristics of the cavity material differ significantly from that of the material filling the gap (in our case, a vacuum). For low cavity absorptions ($|\mathrm{Im}\,\varepsilon_1| \lesssim 1\text{-}10$) $\bar{e}_\omega$ differs little from e_ω^0, the radiation coefficient of an isolated sphere into a vacuum [1], while for high absorptions this difference is quite marked, a reduction occurring: $\bar{e}_\omega < \varepsilon_\omega^0$. The periodicity of $e_\omega(\gamma)$ and the presence of quite intense maxima and minima in heat exchange at a given frequency x must be considered in practical applications for optimization of spatial arrangements, especially when the role of spectral flux components is great. As for the total integral of the flux over frequency, it retains the dimensional effect of intensified heat transport in narrow gaps, while the intense periodicity with respect to γ apparently disappears because of smoothing of the Planck functions upon integration. Nevertheless, existence of individual isolated shallow minima and slight maxima in the total radiation is possible as $\gamma \to 1$ or there may be a rapidly damping oscillating character to e_{int} as $\gamma \to \infty$.

3. In the Rayleigh region $x \ll 1$ and for any B, ε_1 in complete correspondence to the above explanation heat exchange through the spherical gap takes on resonant attributes — for certain ratios between λ and r_c abrupt collective excitations of heat transport intensity develop. The numerical data permit proposal of a simple expression for the positions of these resonances, namely

$$2r_c \approx p\lambda/2 \tag{6}$$

or

$$x\gamma \approx p\pi/2,\ p = 2,\ 3,\ 4,\ \ldots, \tag{7}$$

where the approximate equality is replaced by ever more accuracy with increase in p, namely: the first resonance with $p = 2$ appears at $x\gamma \approx 2.6\text{-}2.7$, the second ($p = 3$) at $x\gamma \approx 4.4\text{-}4.5$, the third, at $x\gamma \approx 6.0\text{-}6.1$, the fourth, at $x\gamma \approx 7.6\text{-}7.7$, the fifth, at $x\gamma \approx 9.2\text{-}9.3$, etc. The resonances noted in Fig. 2 for $x < 1$ are produced by excitation in Eq. (3) of only the fundamental $m = 1$, while the excitations of other harmonics will also obey Eqs. (6), (7) but are significantly sharper and less intense. In essence Eqs. (6), (7) are matching conditions, upon satisfaction of which the greatest interference effect develops in a cavity containing a point Rayleigh particle, if the cavity diameter consists of an integral multiple of half or full wavelengths of the thermal modes. The mode with $2r_c \approx \lambda/2$ disappears due to the "thermal coat" effect of the sphere and cavity [1].

A noteworthy property of the collective heat exchange resonances in the spherical gap is the fact that the intensity of the excitations in Eqs. (6), (7) for the series with $p = 2k$ and $p = 2k + 1$, $k = 1, 2, 3, \ldots$, will depend intensely on the electromagnetic parameters of the cavity and sphere (in our calculations, upon ε_1 and B). For small values of B ($B \lesssim 10$) the intensity of excitations of both series is identical for any ε_1, while for higher B values ($B > 10$) only the first series is more intensely excited, and if $|\varepsilon_1|$ is small only the series $p = 2k$ is excited, while if $|\varepsilon_1| \gg 1$ the series with $p = 2k + 1$ dominates; for $|\varepsilon_1| \sim 1$ both series are practically unexcited. Thus, by changing the cavity absorption parameters the resonant spectral structure of radiant heat exchange can be retuned.

We will also note that in contrast to the behavior of resonant peaks in the radiation of a particle lattice [6], in the case of heat exchange through a spherical gap all resonances do not have an attenuating branch, i.e., at the resonance points only amplification of radiation occurs.

In conclusion we note that the value of the precise form of Eq. (3) for the spectral components of the radiation flux in principle permits finding the total flux by simple numerical integration. However performing such detailed calculations requires a high speed wide bus computer.

NOTATION

ω, frequency; λ, wavelength; c, speed of light; k_B, Boltzmann's constant; T, temperature; $\hbar$, Planck's constant; r, radius; ε, dielectric permittivity; μ, magnetic permeability; κ, conductivity; P_ω, spectral thermal flux; e_ω, spectral radiation coefficient of sphere into cavity; x, diffraction parameter; γ, ratio of cavity radius to particle radius; i, imaginary unit; j, n, $h = h^{(2)}$, spherical Bessel functions of the first, second, and third sorts.

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BEHAVIOR OF MONODISPERSED METAL PARTICLES IN VARIOUS MEDIA

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Processes determining the final properties of monodispersed metallic microgranules are considered.

At present the problem of producing monodispersed metallic systems with special properties is of great practical importance. One of the methods used to produce such systems is high speed cooling of dispersed materials in order to amorphize them, thus producing special properties. Amorphous metal structures are widely used to produce dispersed systems with a characteristic dimension of ~10^{-5} m, and are created at cooling rates of 10^4-10^6 K/sec, which can be achieved by interaction of the objects to be cooled with gaseous or solid media.

In analyzing factors which lead to high cooling rates, we must note the following major ones: contact area, temperature difference between cooling surface and surface being cooled, and thermal conductivity of the material. For a moving metallic microparticle its velocity and temperature are important characteristics controlling the cooling rate (since they influence the contact area), so that it is important to provide a correct mathematical description of processes in order to model the behavior of high temperature dispersed metal systems upon their interaction with various media.

The present study will attempt to consider the possibiity of amorphization of metallic dispersed systems with characteristic dimensions of 10^{-4} m. To solve this problem we will analyze the motion of high temperature ($T \geqslant 1000$—$2000\,^\circ$C) metal (Cu, Mo) microparticles in air and their cooling rate on a copper substrate. The high temperature metal particles were generated by the pulsed arc method [1]. The coefficient of microgranule variation over size did not exceed 5%. Figure 1 shows copper microgranules 140 μm in radius, obtained by this method.

The change in temperature (brightness method) and velocity of the newly formed microgranules was determined as they moved along the vertical axis. The microparticle velocity $v = h/t_r$ was determined by photographic recording of particle motion through a chopper with

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